Fast minimum-bounding rectangle estimation using angular histogram

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Abstract A fast and efficient minimum-bounding rectangle estimation of an image object is proposed for industrial applications such as object inspection and classification. To detect the minimum-bounding rectangle of an object, we estimate the directions of its major and minor axes by using the proposed angular histogram with n sample edge points, instead of using the entire boundary. The final bounding rectangle is that which results in the minimum error when the four vertices are rotated just twice amongst three candidate bounding rectangles. Experiments are carried out to evaluate the performance of the proposed method, and show that it can detect the minimum-bounding rectangle of an object in a fast and efficient manner.

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1. Introduction

The detection of the minimum-bounding rectangle of an image object is an important function for vision-based industrial applications such as the estimation of an object's size, or for the inspection and assembly of objects. The main work of these vision-based application systems is to find the minimum-bounding rectangle, which is the smallest rectangle that contains every points in the object. Figure 1 shows the setup of our box inspection system and bounding rectangle estimation procedures involved in the proposed algorithm.

![Image of box inspection system setup]

**Figure 1.** Setup of the optical device and steps for bounding rectangle estimation.

To determine the minimum-bounding rectangle of an object, it is essential to accurately estimate the major and minor axes. In previous research, the major axis was estimated using all of the pixels included in an object\(^3-4\), or using only the pixels at the edge of an object\(^5-6\). Toussaint\(^1\) computed the minimum-bounding box based on a theorem that the minimum-area rectangle enclosing a convex polygon has a side that is collinear with one of the edges of the polygon. Suesse and Voss\(^7\) proposed a region-based fitting algorithm using a method of normalization and using the moment of area. Chaudhuri and Samal\(^8\) used the boundary points of a blob or region, instead of the internal points. Using these edge points, the directions of the major and minor axes were determined using a least-squares technique. From the orientation of the object, the four vertices of the bounding rectangle were computed. Chaudhuri\(^9\) assumed that the center of the ellipse or circle coincides with the centroid of all the border points of the object. The extension of this idea to 3-D fitting for spheres, spheroids, and ellipsoids was also proposed.

However, since the boundary of the objects used in industrial applications is rough and irregular owing to noise and uneven illumination, we need to construct a robust estimation algorithm that overcomes the above difficulties. Furthermore, sufficiently fast real-time processing is necessary for industrial applications such as object inspection and classification. In this paper, we propose a fast and accurate minimum-bounding rectangle estimation method using an object's boundary and an angular histogram that is obtained from sampling various points along its boundary.
2. Major axis estimation using angular histogram

After image binarization using global thresholding\(^5\), we first extract the boundary of an object using a contour tracing algorithm\(^6\), and compute its centroid using the center of mass method. This method uses the boundary points of the object, instead of the inner points, to reduce the processing time and complexity. Once the centroid \((x_c, y_c)\) of the object has been computed, we can estimate the directions of the major and minor axes using the proposed angular histogram with \(n\) sampled edge points, instead of using the sum of the square of the distances from the centroid to all boundary points.

Given that an object \(A\) has \(N\) boundary points, only \(n\) edge points \((e_i)\) are uniformly sampled from this set of boundary points, which we denote as the set \(E = \{e_1, e_2, e_3, \ldots, e_n\}\). Then, pair-wise edge points \((\lambda_i)\) are extracted in a clockwise direction by pairing up two neighboring edge points. These are denoted as the set \(\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{n-1}\}\), as shown in Figure. 2a. Each element \(\lambda_i\) of the set \(\Lambda\) consists of the following four components:

\[
\lambda_i = (e_i, e_{i+1}, \Delta x_i, \Delta y_i)
\]

where \(\Delta x_i\) and \(\Delta y_i\) are displacements in the horizontal and vertical directions as shown in Figure. 2a, respectively. From the set \(\Lambda\), the orientations of all pair-wise edge points are computed as follows:

\[
\theta_i = \arctan(\Delta y_i / \Delta x_i)
\]

(1)

The radian \(\theta\) is converted into an angle and the angular histogram \(A_{\text{histo}}[\theta]\) is produced as described below:

\[
\text{If } (\theta_i \leq 180) \quad \{ A_{\text{histo}}[\theta_i] ++ ; \}
\]

\[
\text{Else } \{ \theta_i = \theta_i - 180; A_{\text{histo}}[\theta_i] ++ ; \}
\]

(2)

In equation (2), we shift the angle from 0 to 180\(^\circ\) if it is above 180\(^\circ\), because the angles are symmetric with respect to the \(x\)-axis. After computing the angular histogram, the direction of the major axis is determined using the following formula, and as shown in Figure. 2b.

\[
\text{Major angle} = \arg \max_{\theta=1-180} (A_{\text{histo}}[\theta])
\]

(3)

where the \(\max\) operation returns an angle \((\theta)\) that has the maximum frequency of occurrence. From the direction of the major axis, the direction of the minor axis is automatically computed as being perpendicular to it. The major and minor axes pass through the centroid with a slope of \(\theta\) degrees.

Once the two axes have been computed, the upper and lower edge points are computed with respect to the major and minor axes from the following equation [3]:

\[
V = (e_i^y - y_c) - \tan \theta (e_i^x - x_c)
\]

(4)
where \( \mathbf{V} > 0 \) indicates that edge point \( e_i \) is an upper edge point with respect to the furthest lower and upper edge points, by drawing parallel lines to the major and minor axes. By solving for the intersections of the four lines [3], the four vertices of the bounding rectangle can be obtained using equation (5).

\[
\begin{align*}
(t_l_x) &= \frac{x_1 \tan \theta + x_3 \cot \theta + y_3 - y_1}{\tan \theta + \cot \theta}, \\
(t_l_y) &= \frac{y_1 \cot \theta + y_3 \tan \theta + x_3 - x_1}{\tan \theta + \cot \theta} \\
(tr_x) &= \frac{x_1 \tan \theta + x_4 \cot \theta + y_4 - y_1}{\tan \theta + \cot \theta}, \\
(tr_y) &= \frac{y_1 \cot \theta + y_4 \tan \theta + x_4 - x_1}{\tan \theta + \cot \theta} \\
(bl_x) &= \frac{x_2 \tan \theta + x_3 \cot \theta + y_3 - y_2}{\tan \theta + \cot \theta}, \\
(bl_y) &= \frac{y_2 \cot \theta + y_3 \tan \theta + x_3 - x_2}{\tan \theta + \cot \theta} \\
(br_x) &= \frac{x_2 \tan \theta + x_4 \cot \theta + y_4 - y_2}{\tan \theta + \cot \theta}, \\
(br_y) &= \frac{y_2 \cot \theta + y_4 \tan \theta + x_4 - x_2}{\tan \theta + \cot \theta}
\end{align*}
\]

where \((t_l_x, t_l_y), (tr_x, tr_y), (bl_x, bl_y)\) and \((br_x, br_y)\) are the top left, top right, bottom left and bottom right \((x, y)\) coordinates for bounding rectangle. \((x, y)\) and \((x', y')\) is the upper-lower furthest edge points of object \( A \) with respect to major axis and \((x, y)\) and \((x', y')\) is the upper-lower furthest edge points of object \( A \) with respect to minor axis.

The boundary of an object can be distorted by noise or by irregular illumination. To accurately determine the major axis, \( \mathbf{V} < 0 \) indicates that edge point \( e_i \) is a lower edge point with respect to the major axis, and \( \mathbf{V} = 0 \) indicates that the edge point \( e_i \) lies directly on the major axis. The final vertices of the four corners of the boundary rectangle are computed from the

![Figure 2. Four elements of an edge-point pair (Figure 2-(a)) and the angular histogram that is estimated from all such pairs (Figure. 2-(b)). The angular histogram shows the direction of the major axis to be 74°, as this is the angle that occurs most frequently.](image-url)
direction of the major axis for a target image, the four vertices are rotated just twice, from -1° to +1° in a step of 1°. Matching errors are estimated between the four lines that connect the four vertices and the object’s contour. The final bounding rectangle is computed as being the rectangle which has the minimum error amongst the three candidate bounding rectangles. Figure 3 shows the process of determining the bounding rectangle. The dotted rectangles are the candidate rectangles, and the thick rectangle is the final bounding rectangle.

Figure 3. Rotation of the four candidate vertices from -1° to +1° in steps of 1°. The final bounding rectangle is represented by the solid red line.

3. Experimental Results

To evaluate the accuracy of our proposed algorithm, we compared its performance to that of Chaudhuri and Samal [3], which generally performs better than the other algorithms discussed. The experiments were performed using five box images and were taken using a CCD camera in a real work environment with an 800 × 600 pixel image size. As there is no standard method for evaluating the quality of a fitted bounding box, we defined the ratio of empty pixels (ROE) as a metric for comparison. The ROE is the ratio of empty pixels divided by the number of real pixels of the object within the detected minimum-bounding rectangle.

Table 1. Comparison between the ROE performance of the minimum-bounding rectangle estimation methods: our proposed technique and that of Chaudhuri and Samal [3].

<table>
<thead>
<tr>
<th>Objects (Detection Results)</th>
<th>Ratio of Empty Pixels(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Proposed</td>
</tr>
<tr>
<td>5.55</td>
<td>6.36</td>
</tr>
<tr>
<td>2.81</td>
<td>4.25</td>
</tr>
<tr>
<td>2.93</td>
<td>3.25</td>
</tr>
<tr>
<td>10.17</td>
<td>14.34</td>
</tr>
<tr>
<td>16.41</td>
<td>16.6</td>
</tr>
<tr>
<td>Average</td>
<td>7.57</td>
</tr>
</tbody>
</table>

As shown in Table 1, our proposed method results in an improvement in detection performance. Overall, our proposed approach outperformed Chaudhuri and Samal’s method [3] with an average ROE of 7.57% compared to 8.96%, respectively. In addition to improving the ROE, the
processing time for computing the minimum rectangle is approximately 1.06 times faster using our method versus that of Chaudhuri and Samal [3], taking 0.093 s and 0.099 s, respectively, using the same Intel® Core™ i7 PC running a Windows® 7 operating system.

4. Conclusions

A fast and accurate minimum-bounding rectangle estimation method has been proposed using an object's boundary and an angular histogram. To estimate the directions of the major and minor axes, we have proposed using an angular histogram with n sampled edge points. The final bounding rectangle can be computed as that which has the minimum error when the four vertices are rotated just twice amongst three candidate bounding rectangles. Experiments were conducted to evaluate the performance of the proposed method. These experiments have confirmed that the minimum-bounding rectangle of an object can be computed in a fast and efficient manner.

References

[1] G. Toussaint, Solving geometric problems with the rotating calipers, *IEEE Int. Conf. on MELECON*, Quebec, Canada, 1, 1983.


